# MATLAB Marina: Numerical Differentiation with Noisy Data

**Numerical Differentiation with Noisy Data**

One must be careful when performing numerical operations on noisy data (data with uncertainties or data with measurement error). The noise or error associated with measured or uncertain data is often small in magnitude but rapidly changing.

Differentiation tends to magnify the effect of error or noise whereas integration tends to reduce the effect of error or noise. Differentiation which gives the rate of change of something, magnifies things that vary rapidly even if the magnitude of the variation is small. The derivative of the error or noise in data may be larger in magnitude than the derivative of the underlying function of the data.

Figure 1 shows a MATLAB program that generates a noisy signal and determines the numerical derivative. The underlying function is a unit amplitude cosine of 250Hz. The additive noise is cosines of 60Hz and 120Hz with amplitudes uniformly distributed over the range -0.05 to 0.05 and -0.03 to 0.03. Figures 2a and 2b show plots of the noisy data and the approximate derivative of the noisy data.

```
% 250Hz cosine with additive noise modeled as 60Hz and 120Hz
% cosines with varying amplitude
T = 1/250;
t = 0:T/20:5*T;
% unit amplitude cosine of 250Hz
g = 1.0*cos(2*pi*250*t);
% noise, random amplitude 60Hz and 120Hz cosines
noise60 = (-0.05 + 0.1*rand(1,length(t))).*cos(120*pi*t);
noise120 = (-0.03 + 0.06*rand(1,length(t))).*cos(240*pi*t);
% data plus noise
gNoisy = g + noise60 + noise120;

% numerical derivative of the noisy data
dgNoisy = diff(gNoisy)./diff(t);
```

Figure 1. Program for Numerical Derivative of Noisy Data

The noise in the data (Figure 2a) is most noticeable at the peaks. It is barely discernable in between the positive and negative peaks.

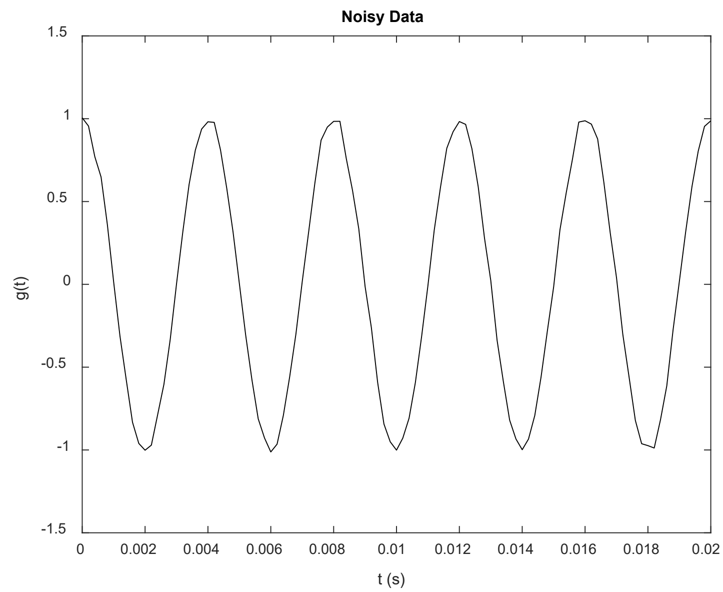


Figure 2a. Noisy Data

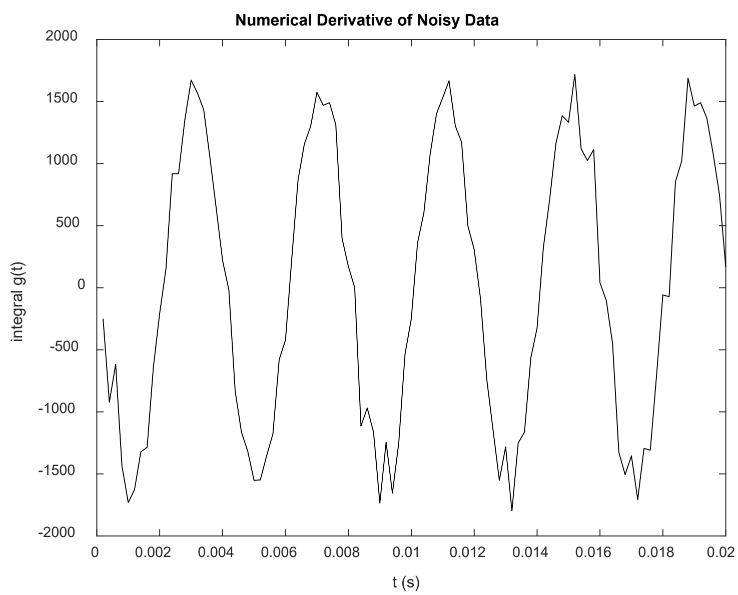


Figure 2b. Numerical Derivative of Noisy Data

The derivative of the noisy signal, shown in Figure 2b, is approximately a sine of 250Hz phase shifted by 180 degrees. (phase shift of 180 degrees corresponds to a negation). The amplitude is much larger than one might expect, but recall that $\frac{d}{dt}\cos(\omega t) = -\omega\sin(\omega t)$, so the amplitude of the sine wave should be scaled by $\omega$ times the amplitude of the noisy signal. The noise in the numerically differentiated signal is very noticeable even between the peaks.

In this example the magnitude of the noise was 8% (5% plus 3%) or less of the original signal. The magnitudes were multiplied by a cosine of 60Hz or cosine of 120Hz which for most points reduced the magnitude of the noise. The average noise magnitude for the signal is around 2.3%.

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